

Clock-transport synchronisation for neutrino time-of-flight measurements

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Abstract

A method to synchronise, at the sub-nanosecond level, clocks used for neutrino time-of-flight measurements is proposed. Clocks situated near the neutrino source and target are compared with a moveable clock that is transported between them. The general-relativistic theory of the procedure was tested and verified in an experiment performed by Hafele and Keating in 1972. It is suggested that use of such a synchronisation method may contribute to a precise test of the Sagnac effect —a measured velocity greater than c — for neutrinos of the proposed LBNE beam between Fermilab and the Homestake mine.

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The aims of the present paper are twofold, firstly to present an analysis of clock transport synchronisation as a possible calibration method for time-of-flight measurements in two existing neutrino beams [1, 2] and one proposed one [3] and secondly to point out the interest of such measurements, at the level of 1ns accuracy, as a test of the Sagnac effect for neutrinos —that the measured velocity is not expected to be exactly equal to the speed, c , of light in vacuum [4, 5]. The clock transport synchronisation method is not proposed as a stand-alone one but rather as a cross-check of continous methods such as GPS Common-View GPS [6] as used for the published neutrino time-of-flight measurements or Two Way Satellite Time Transfer [7] which have a comparable precision ($\simeq 1$ ns or better) than the method proposed here.

In the region of the Earth, the proper time interval, $d\tau$, of a clock is related to the interval of ‘coordinate time’, dt , by the Schwarzschild metric equation [8, 9]:

$$d\tau = \left[1 + \frac{2\phi_E}{c^2} - \frac{1}{c^2} \left(\frac{v_r^2}{1 + \frac{2\phi_E}{c^2}} + v_\theta^2 + v_\phi^2 \right) \right]^{\frac{1}{2}} dt \quad (1)$$

where v_r , v_θ and v_ϕ are components of the velocity of the clock in a spherical polar coordinate system with origin at the centre of the Earth, and polar axis in the south-north direction, fixed in the Earth Centered Inertial (ECI) frame. The ECI frame is a non-rotating inertial frame instantaneously comoving with the centroid of the Earth. The coordinate axes of the ECI frame therefore have fixed directions relative to the Celestial sphere. The quantity ϕ_E is the gravitational potential of the Earth, which, on the assumption of a spherical Earth is given as: $\phi_E = -GM_E/r$ at distance r from the centre of the Earth, where M_E is the mass of the Earth. As can be seen by setting $\vec{v} = 0$,

$\phi_E = 0$ in (1), coordinate time is the proper time of any clock at rest in the ECI frame sufficiently far from the Earth that all gravitational effects of the latter may be neglected. In applying (1) it is assumed that the spatial variation of the gravitational fields of the Sun, Moon and other members of the Solar System, that give rise to tidal effects, may be neglected, as well as the rotation of the Earth around the Sun that changes the angular velocity of a rotating, Earth-fixed, frame relative to the ECI frame. These are the same approximations that are made in applications of (1) to clocks in the satellites of the GPS system [10, 11].

For a clock at rest on the Surface of the Earth $v_r = v_\theta = 0$ so that (1) simplifies to:

$$d\tau = \left[1 + \frac{1}{c^2} (2\phi_E - \Omega^2 R_{xy}^2) \right]^{\frac{1}{2}} dt \quad (2)$$

where R_{xy} is the distance of the clock from the axis of rotation of the Earth and Ω is the angular velocity of the latter relative to the ECI frame. Due to its rotation, the Earth has the form of an oblate spheroid, the surface of which is known as the geoid. Locally, in the oceans, the plane of the geoid is parallel to the surface of the sea.

It was shown by Cocke [12] that the distortion of the Earth from a spherical shape modifies the external gravitational potential of the Earth in such a way that, to a very good approximation, the quantity $2\phi_E - \Omega^2 R_{xy}^2$ in (2) is constant for all points on the geoid so that identical clocks at the poles or on the Equator will be observed, from the ECI frame, to run at the same rate^a.

If a clock moves, at fixed r , relative to the surface of the Earth, with speed \vec{v} , the elapsed proper time interval $\Delta\tau$ is given by integrating (1):

$$\Delta\tau = \int \left[1 + \frac{1}{c^2} (2\phi_E - (\vec{v}_\Omega + \vec{v})^2) \right]^{\frac{1}{2}} dt \simeq \int \left[1 + \frac{1}{c^2} \left(\phi_E - \frac{1}{2}(\vec{v}_\Omega + \vec{v})^2 \right) \right] dt \quad (3)$$

where \vec{v}_Ω is the speed, in the ECI frame, of an object, at the same position as the clock, but at rest relative to the Earth, and where, in the last member, only terms linear in ϕ_E/c^2 and quadratic in v/c are retained.

As suggested by Hafele [15, 16] the correctness of the relation (3) was verified in an experiment performed by Hafele and Keating (HK) [17] in 1972. An array of caesium-beam atomic clocks was flown around the Earth in commercial airliners in West to East (W-E) and East to West (E-W) directions. They were compared, before and after the flights, with reference Earth-bound clocks at the U.S. Naval Observatory. In agreement with prediction, time interval differences, relative to the reference clocks, of 273ns for the W-E and -59ns for the E-W flights were observed [17].

In the application of Eq. (3) suggested, in the present work, to synchronise clocks for time-of-flight measurements, comparisons are made between the epochs recorded by similar clocks (i.e. ones running at the same frequency), one at the position of the source, and the other at the position of the detector, of the particles. If the clocks are placed on the geoid they will be observed from the ECI frame to run at the same rate. As will be shown, the clocks can be synchronised by comparing their epochs with that of a third similar clock that is transported between them. The single fixed reference clock of

^aNot as stated in Einstein's 1905 special relativity paper [13], slower at the equator. See [14] for a discussion of this point.

the HK experiment is therefore replaced by two similar clocks at different, but known, spatial locations. In order to make this comparison the proper time intervals $\Delta\tau_0$, $\Delta\tau$ recorded by the Earth-fixed and moving clocks, respectively, for the same interval, Δt , of coordinate time are calculated using Eq. (3) taking account of the variation of the values of ϕ_E , \vec{v}_Ω and \vec{v} along the path followed by the moveable clock. For the purposes of the calculation presented here it is assumed that the Earth is spherical; in actual experimental applications the actual shape of the geoid [11] should be taken into account.

In a practical realisation of the synchronisation method suggested here the clock might be equipped with a GPS receiver and associated hardware to record the precise position of the clock in the Earth-fixed frame at known epochs during the transport so that the integral in Eq. (3) can be accurately evaluated. In the present work the dependence of $\Delta\tau$ on the parameters of the clock trajectory is studied within a simple model where only the mean speed and altitude of the transport are considered. This is sufficient to quantitatively demonstrate the predictions for $\Delta\tau$ given by different choices of transport trajectory for the three neutrino time-of-flight experiments that are considered.

Considering the specific case of neutrino time-of-flight measurements in the CNGS neutrino beam [2, 22] the proper time intervals recorded by reference clocks at CERN ($\Delta\tau_0^C$) and Gran Sasso ($\Delta\tau_0^{GS}$), assumed to lie on the geoid, are given by (3), to first order in ϕ_E and second order in $\beta = v_\Omega/c$ as

$$\begin{aligned}\Delta\tau_0 &= \Delta\tau_0^C = \int \left[1 + \frac{1}{c^2} \left(\phi_E^C - \frac{1}{2}(\vec{v}_\Omega^C)^2 \right) \right] dt \\ &= \Delta\tau_0^{GS} = \int \left[1 + \frac{1}{c^2} \left(\phi_E^{GS} - \frac{1}{2}(\vec{v}_\Omega^{GS})^2 \right) \right] dt\end{aligned}\quad (4)$$

since $\phi_E - (\vec{v}_\Omega)^2/2$ is invariant on the geoid. The proper time interval $\Delta\tau$ recorded by a clock transported from CERN (C) to Gran Sasso (GS) during the interval Δt of coordinate time is, from (3) and (4):

$$\Delta\tau = \Delta\tau_0 + \int_C^{GS} \left[\frac{GM_E h}{c^2 R^2} - \frac{1}{2c^2} [(\vec{v}_\Omega)^2 - (\vec{v}_\Omega^C)^2 + v^2 + 2\vec{v}_\Omega \cdot \vec{v}] \right] dt \quad (5)$$

where h is the altitude above the Earth's surface. Using a mean value approximation:

$$\int f(t) dt \simeq \langle f(t) \rangle \int dt$$

to evaluate the integral in (5), and assuming transport at constant v , gives:

$$\frac{\Delta\tau - \Delta\tau_0}{\Delta t} \simeq \frac{\Delta\tau - \Delta\tau_0}{\Delta\tau_0} \equiv \frac{\delta\tau}{\Delta\tau_0} = \frac{GM_E \langle h \rangle}{c^2 R^2} - \frac{1}{2c^2} [\langle (\vec{v}_\Omega)^2 \rangle - (\vec{v}_\Omega^C)^2 + v^2 + 2\langle \vec{v}_\Omega \rangle \cdot \vec{v}] \quad (6)$$

Since the values of $\delta\tau$ in (6) are of the order of a few nanoseconds precise modelling of the altitude and speed of the travelling clock in order to evaluate the integral in (5) is not required. For example a 5% uncertainty in the evaluation of the integral gives only 0.05ns uncertainty in $\delta\tau$ when $\delta\tau = 1$ ns.

Introducing a primed, Earth-centered, Earth-fixed, polar coordinate system with north-pointing polar axis, $\phi' = 0$ at the prime meridian and λ' the angle of latitude, the coordinates of CERN are [18]: $\lambda'_C = 46.05^\circ$, $\phi'_C = 6.08^\circ$ and of Gran Sasso are: $\lambda'_{GS} = 42.3^\circ$,

$\phi'_{\text{GS}} = 13.58^\circ$. If \hat{i}', \hat{j}' and \hat{k}' are unit vectors along the x' ($\lambda' = 0, \phi' = 0$), y' ($\lambda' = 0, \phi' = 90^\circ$) and z' axes of the corresponding right-handed Cartesian coordinate system, the spatial separation of the neutrino source at CERN from the OPERA detector at Gran Sasso is $d = \sqrt{(\Delta x')^2 + (\Delta y')^2 + (\Delta z')^2}$ where:

$$\Delta x' = R(\cos \lambda'_{\text{GS}} \cos \phi'_{\text{GS}} - \cos \lambda'_C \cos \phi'_C) = 0.02936R \quad (7)$$

$$\Delta y' = R(\cos \lambda'_{\text{GS}} \sin \phi'_{\text{GS}} - \cos \lambda'_C \sin \phi'_C) = 0.1003R \quad (8)$$

$$\Delta z' = R(\sin \lambda'_{\text{GS}} - \sin \lambda'_C) = -0.0476R \quad (9)$$

The velocity vector \vec{v} parallel to the neutrino beam is then:

$$\vec{v} = v(0.2557\hat{i}' + 0.8734\hat{j}' - 0.4144\hat{k}') \equiv v\hat{v} \quad (10)$$

Also

$$\langle \vec{v}_\Omega \rangle = \Omega R(-\hat{i}' \langle \cos \lambda' \sin \phi' \rangle + \hat{j}' \langle \cos \lambda' \cos \phi' \rangle) = \Omega R(-0.1236\hat{i}' + 0.7047\hat{j}') \quad (11)$$

so that

$$\langle \vec{v}_\Omega \rangle \cdot \vec{v} = 0.5839\Omega R v \quad (12)$$

and

$$\langle (\vec{v}_\Omega)^2 \rangle = \Omega^2 R^2 \langle \cos^2 \lambda' \rangle = 0.5146\Omega^2 R^2 \quad (13)$$

$$(\vec{v}_\Omega^C)^2 = \Omega^2 R^2 \cos^2 \lambda'_C = 0.4815\Omega^2 R^2 \quad (14)$$

Substituting (12)-(14) in (6) with $\Omega R = 465$ m/s gives:

$$\delta\tau = \frac{0.00407}{v} [19.57\langle h \rangle - 7157 - v^2 - 543v] \text{ ns} \quad \text{CERN} - \text{CNGS} \quad (15)$$

with $\langle h \rangle$ in m and v in m/s. Here the relation

$$\Delta t \simeq \Delta\tau_0 = \frac{dC_{\text{GS}}}{v} = \frac{730.53 \times 10^3 \times 1.0022}{v} = \frac{732.14 \times 10^3}{v}$$

is used. The straight line distance d from CERN to the OPERA detector at Gran Sasso [18] is corrected by the factor C_{GS} which accounts for clock transport along a Grand Circle.

A similar calculation for the MINOS experiment [1] in the Fermilab (FNAL) NuMI beam with $d = 732.7$ km, $C_{\text{GS}} = 1.0022$, gives:

$$\delta\tau = \frac{0.00408}{v} [19.57\langle h \rangle + 11222 - v^2 + 280v] \text{ ns} \quad \text{FNAL} - \text{MuMI} \quad (16)$$

Here FNAL is assumed to be at latitude: $\lambda'_F = 41.85^\circ$, longitude: $\phi'_F = -88.29^\circ$ and the MINOS detector in the Soudan mine at $\lambda'_S = 47.81^\circ$, $\phi'_S = -92.24^\circ$.

For the proposed long-baseline FNAL-Homestake neutrino beam [3] LBNE, with $d = 1298$ km, $C_{\text{GS}} = 1.0069$ it is found that:

$$\delta\tau = \frac{0.00726}{v} [19.57\langle h \rangle + 4670 - v^2 + 657v] \text{ ns} \quad \text{FNAL} - \text{LBNE} \quad (17)$$

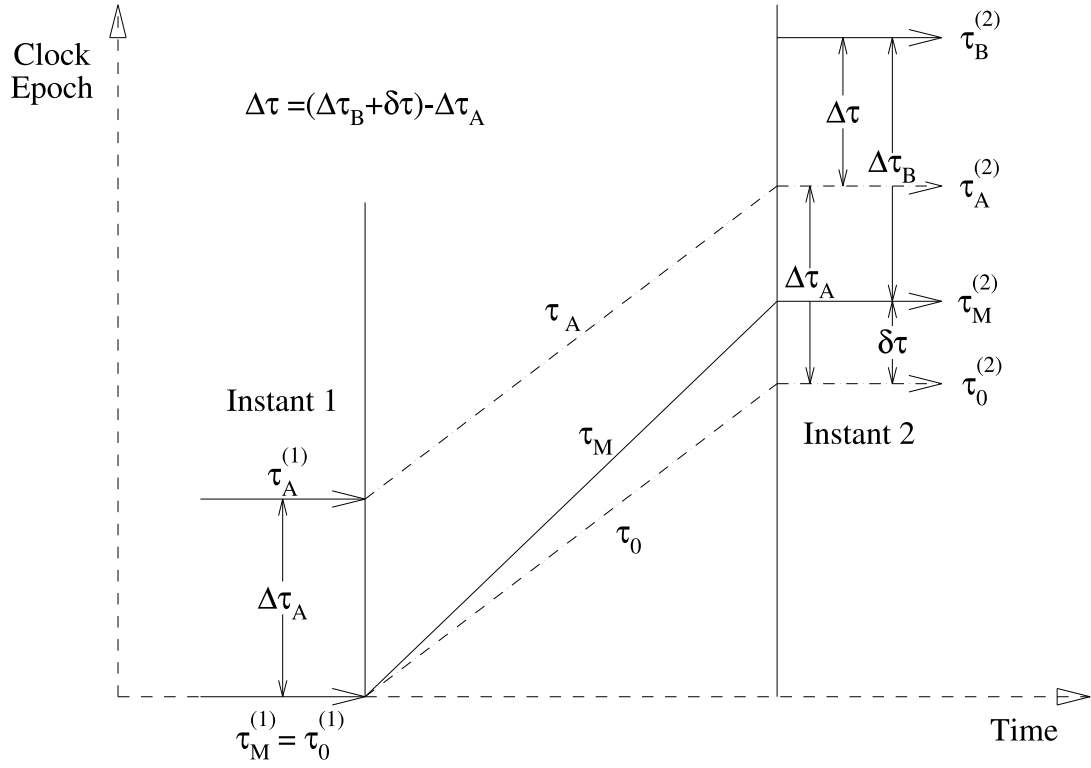


Figure 1: *Procedure to synchronise clocks A and B at different spatial locations on the geoid by comparison with a clock, M, transported at constant velocity between them. See text for discussion.*

$\langle h \rangle$ (km)	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.36551
v_0 (km/h)	305	283	260	234	205	171	129	63	0
$\delta\tau_0$ (ns)	-2.90	-2.85	-2.80	-2.74	-2.67	-2.60	-2.50	-2.35	-2.21

Table 1: Values of v_0 and $\delta\tau_0$ at which $\delta\tau$ has a maximum value, as a function of $\langle h \rangle$, for the CERN-CGNS neutrino beam.

v (km/h)	10	50	100	200	400	600	800	1000
$\langle h \rangle$ (m)								
0	-12.7	-4.4	-3.4	-3.0	-2.9	-3.1	-3.2	-3.4
200	-7.0	-3.2	-2.8	-2.7	-2.8	-3.0	-3.2	-3.4
400	-1.2	-2.1	-2.2	-2.4	-2.6	-2.9	-3.1	-3.3
600	4.5	-0.92	-1.7	-2.1	-2.5	-2.8	-3.0	-3.3
800	10.2	0.22	-1.1	-1.8	-2.4	-2.7	-3.0	-3.2
1000	16.0	1.4	-0.51	-1.5	-2.2	-2.6	-2.9	-3.2

Table 2: Values of $\delta\tau$ in ns as a function of v and $\langle h \rangle$ for the CERN-CGNS neutrino beam.

where the Homestake mine at Lead, South Dakota, has coordinates: $\lambda'_H = 44.33^\circ$, $\phi'_H = -103.83^\circ$ [19].

The procedure to synchronise a clock, A , at the source laboratory, registering an epoch τ_A with clock B , at the detector laboratory, registering an epoch τ_B is as follows. Before the clock transport, the difference, $\Delta\tau_A$, between the epochs registered by clock A and the moveable clock M are recorded at some instant 1. $\Delta\tau_A = \tau_A^{(1)} - \tau_M^{(1)}$. After the clock transport the difference between the epochs of B and M is recorded at some instant 2: $\Delta\tau_B = \tau_B^{(2)} - \tau_M^{(2)}$. In order to synchronise clock B with clock A the quantity $\Delta\tau \equiv \Delta\tau_B - \Delta\tau_A + \delta\tau$ (see Fig. 1) must be subtracted from the epoch registered by clock B . Note that if all three clocks have the same frequency when at rest in the same frame of reference the differences of epochs $\Delta\tau_A$ ($\Delta\tau_B$) may be recorded at any time before (after) the transport of M , so that the instants 1 and 2 are arbitrary.

Curves of $\delta\tau$ versus v for various values of $\langle h \rangle$ as calculated by (15), (16) and (17) are shown in Figs. 2, 3 and 4 respectively. A feature of all three neutrino beams is the existence, for some values of $\langle h \rangle$, of ‘magic’ values v_M of the velocity for which $\delta\tau$ vanishes (given by the intersection of the curves with the dashed line) and so gives the possibility of a simple and accurate synchronisation procedure, as if the moving clock was equivalent to the ground-based ones. Magic velocity values correspond to exact cancellation of the special-relativistic time dilation effect, which slows the rate of the moving clock as observed in the ECI frame, and the general-relativistic gravitational blue shift which increases its rate. Curves of v_M versus $\langle h \rangle$ for all three beams are shown in Fig. 5. For values of v : $50 \text{ km/h} < v < 1000 \text{ km/h}$ corresponding to road, rail or air transport $\delta\tau$ takes, in all cases, absolute values of a few nanoseconds or less.

The geometry of the CERN-CGNS beam is such that, for small values of $\langle h \rangle$, the curves of $\delta\tau$ versus v have maximum values. The corresponding velocity v_0 and time

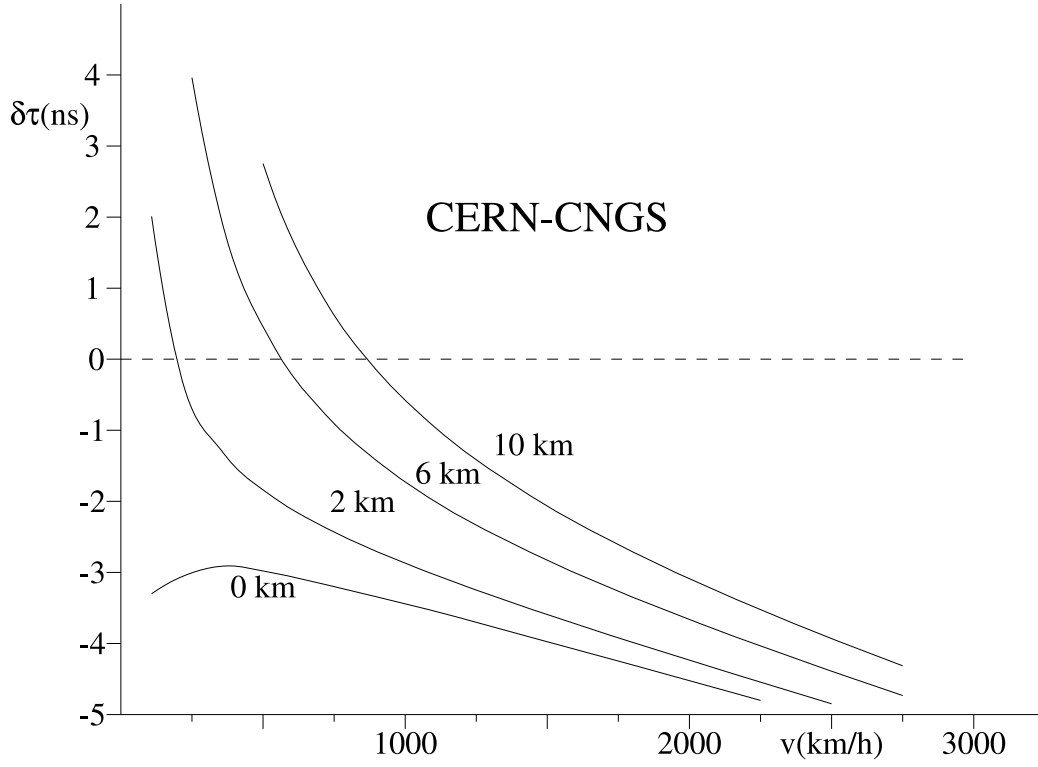


Figure 2: Curves of $\delta\tau$ versus v for different values of $\langle h \rangle$ for the CERN-CNGS neutrino beam. Magic values of v are given by the intersections of the curves with the dashed line $\delta\tau = 0$.

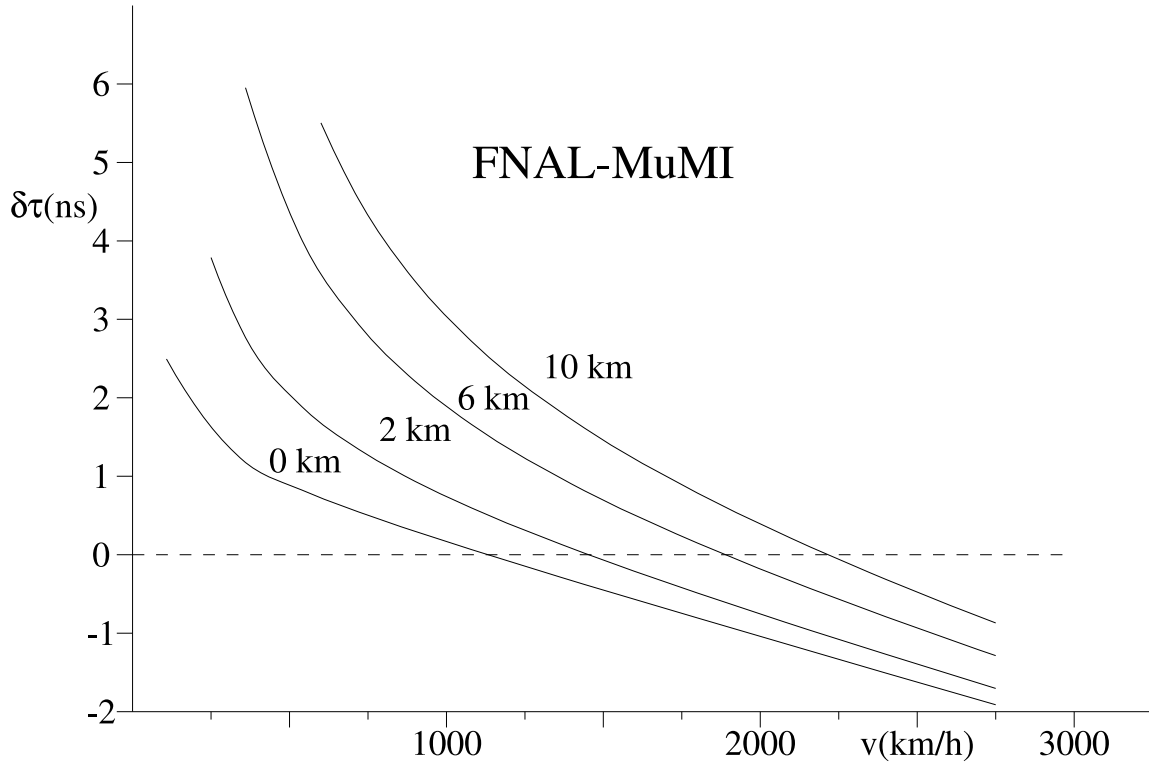


Figure 3: Curves of $\delta\tau$ versus v for different values of $\langle h \rangle$ for the FNAL-MuMI neutrino beam.

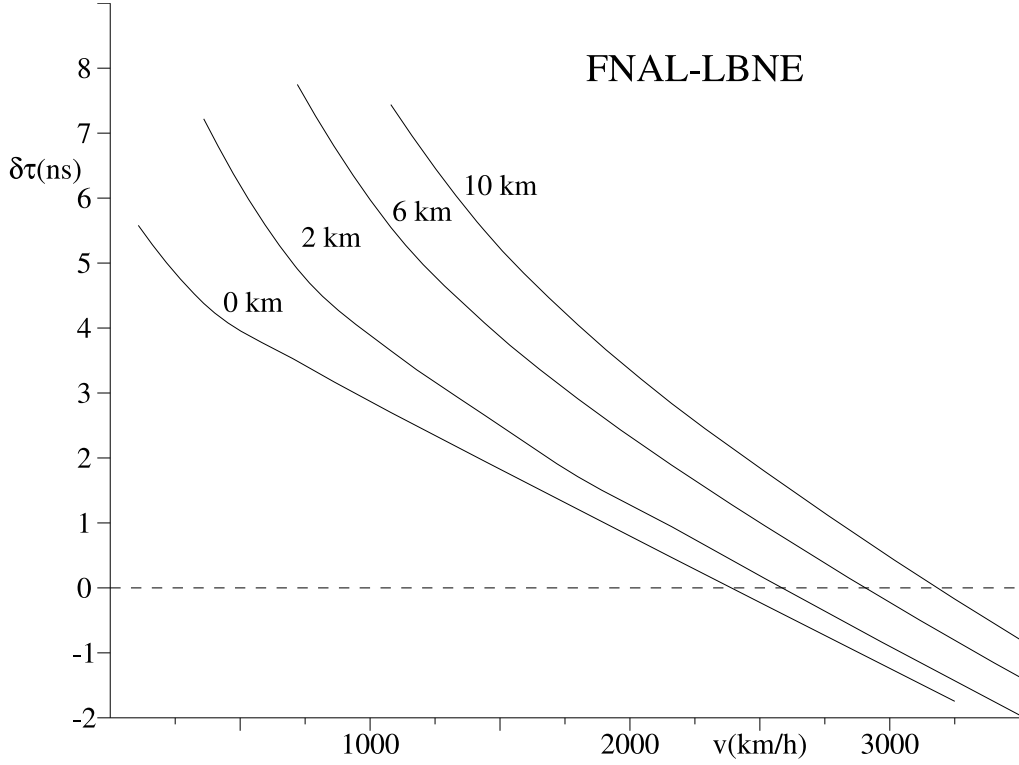


Figure 4: Curves of $\delta\tau$ versus v for different values of $\langle h \rangle$ for the FNAL-LBNE neutrino beam.

interval $\delta\tau_0$ are given by the solution of the equation $d(\delta\tau)/dv = 0$ as:

$$v_0 = \left[\langle (\vec{v}_\Omega)^2 \rangle - (\vec{v}_\Omega^C)^2 - \frac{2GM_E \langle h \rangle}{R^2} \right]^{\frac{1}{2}} \quad (18)$$

$$\delta\tau_0 = -\frac{dC_{GC}}{c^2}(v_0 + \hat{v} \cdot \langle \vec{v}_\Omega \rangle) \quad (19)$$

The largest value of $\langle h \rangle$ for which a maximum value of $\delta\tau$ exists is $\simeq 365$ m corresponding to $v_0 = 0$. The dependence of v_0 and $\delta\tau_0$ on $\langle h \rangle$ is given in Table 1. Over the range $0 \text{ m} < \langle h \rangle < 300 \text{ m}$, $\delta\tau_0$ varies by only $\pm 0.2 \text{ ns}$ about an average value of 2.7 ns . As shown in Table 2 the feature of a weak dependence of $\delta\tau$ on both v and $\langle h \rangle$ persists for $365 \text{ m} < \langle h \rangle < 1000 \text{ m}$ where the curves are monotonic. For example, for $v = 500 \text{ km/h}$ and $\langle h \rangle = 500 \text{ m}$ one percent uncertainties in v and $\langle h \rangle$ give only $\simeq 0.24 \%$ and $\simeq 0.09 \%$ uncertainties, respectively in $\delta\tau$, suggesting that this quantity may be determined with a precision of one tenth of a nanosecond or better. In order to achieve this precision over the typical beam length of $\simeq 1000 \text{ km}$, with $100 \text{ km/h} < v < 1000 \text{ km/h}$ requires a frequency stability of the clock in the range 3×10^{-14} — $3 \times 10^{-13}/\text{h}$, compatible with the typical frequency stability of a precision atomic clock of 1 ns/day .

Since the Great Circle route from CERN to Gran Sasso passes over the Alps a possible strategy to keep h below 1 km to take advantage of the almost constant value of $\delta\tau$ as a function of v and $\langle h \rangle$, as shown in Table 2, is to transport the clocks by air above the Rhone to the Mediterranean, then by a Grand Circle route to Gran Sasso. An alternative strategy is a Grand Circle route directly over the Alps at an altitude of $5\text{-}10 \text{ km}$ at or

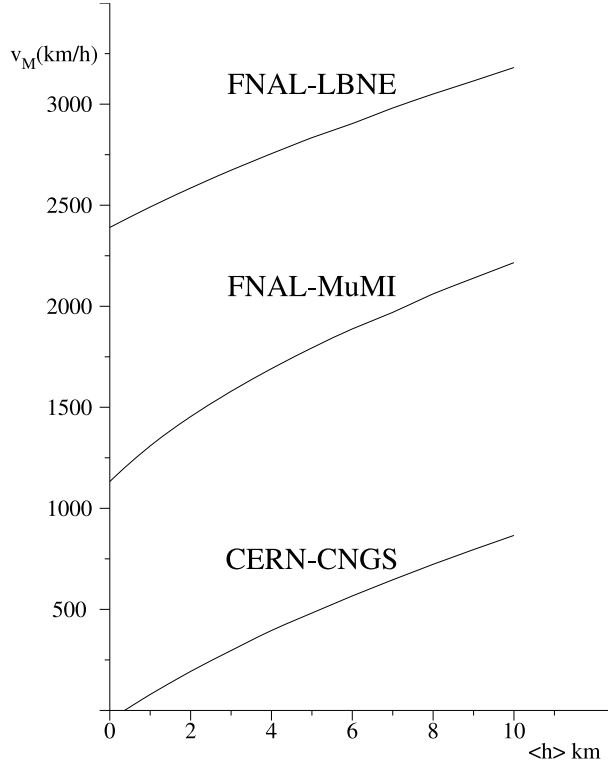


Figure 5: *Magic velocity values as a function of $\langle h \rangle$ for different neutrino beams.*

near the magic value of v , again enabling a precise synchronisation of the local clocks at CERN and Gran Sasso.

For the FNAL neutrino beams:

$$\begin{aligned} \langle (\vec{v}_\Omega)^2 \rangle - (\vec{v}_\Omega^F)^2 &= -11.22 \times 10^3 \quad \text{MuMI} \\ &= -4.67 \times 10^3 \quad \text{LBNE} \end{aligned}$$

Inspection of Eq. (18) shows that no real value of v_0 exists for these beams; the curves of $\delta\tau$ versus v are therefore monotonic for all values of $\langle h \rangle$. As can be seen in Figs. 3 and 4 magic values of v are obtainable by conventional air transport at speeds of 1000-2000 km/h for the MuMI beam but not for LBNE that requires speeds in excess of 2000 km/h.

It is interesting to compare the uncertainty in the clock synchronisation constant $\delta\tau$ with the variation of neutrino times-of-flight due to the Sagnac effect [4, 5, 20, 21], i.e. the effect of the motion of the neutrino target, due to the Earth's rotation, on the observed time-of-flight. For neutrinos with energy much greater than their mass, the relative speed c_r of the neutrino and the target is, at lowest order in the velocity \vec{v}_Ω^T of the target, given by [4, 5]:

$$c_r = c - \hat{v} \cdot \vec{v}_\Omega^T \quad (20)$$

where \hat{v} is a unit vector in the direction from the neutrino source to the target. For the

three neutrino beams considered above it is found that:

$$\begin{aligned}\frac{c_r - c}{c} &= -9.05 \times 10^{-7} && \text{CNGS} \\ &= 4.65 \times 10^{-7} && \text{MuMI} \\ &= 1.10 \times 10^{-6} && \text{LBNE}\end{aligned}$$

The corresponding differences of time-of-flight compared with propagation at speed c are: 2.2 ns [4], -1.14 ns and -4.74 ns respectively, to be compared with the possible uncertainty in $\delta\tau$ of the order of a nanosecond or less. Especially the proposed FNAL-LBNE beam [3] is well adapted to a precise test of the Sagnac effect for neutrinos.

In the case of four experiments which have published neutrino time-of-flight measurements: MINOS [1], OPERA[22], ICARUS[23] and LVD [24] the dominant systematic errors were related to local measurements of the times of production and detection of the neutrinos rather than errors in the synchronisation of the precision master clocks near to the source and target. In the LVD measuremet that has the smallest quoted systematic uncertainty of 3.3 ns on the time-of-flight measurement, only 1 ns is assigned to GPS synchronisation. In order to be sensitive to the Sagnac effect more accurate local timing of neutrino production and detection events than hitherto achieved is required in addition to synchronisation uncertainty at the 1 ns level or better.

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